MCR-10: A Constraint-Based Framework for Mathematical Cognitive Reconstruction

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***Abstract*—This paper introduces MCR-10 (Mathematical Cognitive Reconstruction), a constraint-based framework for reconstructing feasible cognitive models from historical decisions. Unlike psychological or probabilistic approaches, MCR-10 does not attempt to recover a true cognitive architecture or infer unobservable mental states. Instead, it decomposes decisions into evidence-anchored constraints and reconstructs the set of all deterministic cognitive functions consistent with those constraints. Determinism is treated as a modeling stance rather than an empirical claim, and non-uniqueness is acknowledged as irreducible. MCR-10 outputs a minimal, time-bounded feasible set of cognitive models, providing a reproducible, mathematically grounded methodology for computational historical cognition.**

I. Introduction

Reconstructing the cognition of historical agents has traditionally relied on narrative interpretation and psychological inference, both of which introduce unverifiable assumptions about internal mental states. Computational approaches to cognition often rely on probabilistic inference or utility maximization [1], [2], which are unsuitable for historical reconstruction due to limited data and the impossibility of estimating latent variables.

Constraint-based modeling [3], [4] and inverse problem theory [5] provide a more disciplined foundation for reconstructing feasible explanations from incomplete evidence. MCR-10 builds on these traditions by treating cognitive reconstruction as a constraint satisfaction problem rather than a predictive or psychological task.

This paper presents the theory, mathematical formalization, reconstruction algorithm, and pipeline of MCR-10, along with its assumptions and limitations.

II. Theory

MCR-10 is founded on three principles: *constraint-based reconstruction*, *evidence-anchored dimensionality*, and *irreducible non-uniqueness*.

A. Constraint-Based Reconstruction

Every decision is decomposed into constraints reflecting context, incentives, pressures, values, and available options. This aligns with classical constraint satisfaction frameworks [4] and avoids speculative inference.

B. Evidence-Anchored Dimensions

Only dimensions with explicit historical grounding are admissible. This follows the epistemic discipline advocated in historical inference literature [6].

C. Non-Uniqueness as Irreducible

Inverse cognitive reconstruction is inherently underdetermined [5]. MCR-10 therefore outputs a *feasible set* of cognitive models rather than a single clone.

III. Mathematical Formalization

Let the set of documented decisions be:

$$D = \{d_1, d_2, \ldots, d_k\}.$$

Let the evidence-anchored stimulus space be:

$$S = \mathbb{R}^n,$$

where each dimension is historically grounded.

A. Hypothesis Class

Cognitive functions are restricted to a bounded class:

$$f \in \mathcal{F},$$

where $\mathcal{F}$ includes interpretable, finite-complexity functions [7] such as monotone rules, threshold functions, piecewise-linear mappings, or finite-state policies.

B. Feasible Cognitive Models

A model (f, Θ) is feasible if:

$$f(S_i) = d_i \quad \forall i.$$

The feasible set is:

$$C = \{(f, \Theta) : f(S_i) = d_i\}.$$

C. Constraint-Solving Operator

$$\mathcal{S}(D, E) = \{(f, \Theta) : \text{all constraints in } E \text{ satisfied}\},$$

consistent with SAT-style formulations [8].

D. Minimality Metric

$$C^* = \arg\min_{(f,\Theta)\in C} \left(|\Theta| + \lambda \cdot \text{complexity}(f)\right),$$

where $|\Theta|$ is the cardinality of evidence-anchored dimensions and complexity(f) follows minimum description length principles [9].

E. Temporal Cognition

$$C = \bigcup_t C_t,$$

where each C_t corresponds to a time-bounded cognitive regime, consistent with piecewise modeling approaches [10].

IV. Algorithm

Input: Historical decisions D **Output:** Minimal feasible cognitive set C^*

1. Extract and encode decisions.
2. Identify evidence-anchored dimensions.
3. Split decisions into reconstruction and validation sets [11].
4. Perform forward reconstruction when stimuli are known.
5. Perform backward constraint recovery when stimuli are unknown.
6. Construct feasible models using the constraint solver $\mathcal{S}$.
7. Apply minimality metric to obtain C^*.
8. Partition models across temporal epochs.
9. Validate all models on held-out decisions; reject failures.

V. Pipeline

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Historical Data
      ↓
Decision Extraction
      ↓
Evidence-Anchored Dimension Identification
      ↓
Forward Reconstruction ↔ Constraint Recovery
      ↓
Constraint Solver (S)
      ↓
Minimality Metric
      ↓
Temporal Segmentation
      ↓
Validation on Held-Out Decisions
      ↓
Feasible Cognitive Set (C*)
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VI. Conclusion

MCR-10 provides a rigorous, constraint-based methodology for reconstructing feasible cognitive explanations from historical decisions. By combining constraint satisfiability [4], evidence anchoring [6], bounded hypothesis classes [7], and explicit validation [11], MCR-10 offers a reproducible, mathematically grounded foundation for computational historical cognition. Its feasible-set framing acknowledges irreducible non-uniqueness [5] and avoids speculative inference, positioning MCR-10 as a disciplined alternative to narrative interpretation and probabilistic persona simulation.

References

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